Question Paper

Exam Date & Time: 30-Apr-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester End Semester Examination

ENGINEERING MATHEMATICS - IV [MAT 2223]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1)

A) The chance that a doctor A will diagnose the disease correctly is 60%. The chance that the patient of A will die after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. If a patient of A dies. What is the probability that his disease was diagnosed correctly? (3)

B) Find the mean and variance of Binomial distribution. (3)

C) The diameter of an electric cable, say X is assumed to be a continuous random variable with pdf (4)

$$f(x)=\begin{cases} kx(1-x), & 0\leqslant x\leqslant 1 \\ 0, & otherwise. \end{cases}$$

i. Find the value of k.

ii. Compute the probability $P\left(X\leq \frac{1}{2} \mid \frac{1}{3}<X<\frac{2}{3}\right)$..

iii. Compute $E(X)$.

2)

A) The heights of 500 soldiers are found to have normal distribution, of them 258 are found to be within $2\ cm$ of the mean height of $170\ cm$. Find the standard deviation. (3)

B) If X, Y and Z are uncorrelated random variables with standard deviations 5, 12 and 9 respectively. If $U=X+Y$ and $V=Y+Z$. Evaluate the correlation coefficient between U and V. (3)

C) Suppose that the joint pdf of (X, Y) is given by (4)

$$f(x,y)=\begin{cases} e^{-y}, & x>0,\ y>x \\ 0, & otherwise. \end{cases}$$

i. Find the marginal pdf of X and Y.

ii. Evaluate $P(Y > X)$.

3) A) If the probability distribution function of X is given by $f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & otherwise. \end{cases}$ Find the pdf of $Y = 3X + 1$.. (3)

B) Suppose that the two-dimensional random variable (X, Y) is uniformly distributed over the region whose vertices are $(1, 0), (0, 1), (-1, 0)$ and $(0, -1)$. Find the marginal pdf of X and Y. (3)

C) Suppose an urn A contains 2 white marbles and urn B contains 4 red marbles. At each step of the process, a marble is selected at random from each urn and the two marbles selected are interchanged. Let X_n denote the number of red marbles in urn A after n interchanges. (4)

i. Find the transition matrix.

ii. What is the probability that there are 2 red marbles in urn A after 3 steps.

4) A) Solve by graphical method Maximize $Z = 5x_1 + 4x_2$ subject to (3)

$$6x_1 + 4x_2 \le 24,\ \ x_1 + 2x_2 \le 6,\ \ x_1 - x_2 \le 1,\ \ x_1 \le 2,\ \ x_{1,} x_2 \ge 0.$$

B) Iterate three steps for minimum of $f(x) = x_1^2 + 3x_2^2$ starting from $X_0 = (4, 2)$ using steepest descent method. (3)

C) Find the solution for LPP using Simplex method (4)

Maximize $Z = 3x_1 + 2x_2$ subject to $x_1 + x_2 \le 4,\ \ x_1 - x_2 \le 2,\ \ x_1, x_2 \ge 0.$

5) A) Find the directional derivative of $\varnothing = 4e^{2x - y + z}$ at the point $(1, 1, -1)$ in the direction towards the point $(-3, 5, 6)$. (3)

B) Find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(2, -1, 2)$. (3)

C) Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is a conservative force field. Find the Scalar potential. (4)

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