Question Paper

Exam Date & Time: 14-Jun-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH END SEMESTER MAKEUP EXAMINATIONS, JUNE 2024

ENGINEERING MATHEMATICS - IV [MAT 2224]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates:Missing data may be suitably assumed

1) Solve the ODE $xy'' + y = 0,\ y(1) = 1,\ y(2) = 2; h = 0.25$ using finite difference method. (4)

A)

B) With h = $\frac{1}{3}$, Solve the Poisson's equation (3)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -81xy;\ 0 < x < 1; 0 < y < 1;\ u(x,1) = u(1,y) = 100;\ u(x,0) = u(0,y) = 0$$

using finite difference method.

C) Solve the heat equation $32\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2};\ 0 < x < 1;\ t > 0;\ h = 0.25$; (3)

$u(x,0) = u(0,t) = 0;\ u(1,t) = t$ up to 4 time steps by taking $\lambda = \frac{1}{2}$.

2) The coefficients a, b, c of the quadratic equation $ax^2 + bx + c = 0$ are determined by throwing a dice three times. (4)

A) Find the probability that the roots are real.

B) Suppose a random variable X has probability mass function (3)

$P\{X = r\} = k(1-\beta)^{r-1}, 0 < \beta < 1, r = 1,2,3, \ldots\ldots\ldots\ldots$. Determine the constant k. For what value of r, $P\{X = r\}$ is largest.

C) A student takes a multiple choice test consisting of 2 problems. The first one has 3 possible answers and the second one has 5. The student chooses at random one answer as the right answer for each of the two problems. Let X denote the number of right answers of the student. Find mean and variance. (3)

3) Two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and (4)

A) $y = 11.64 - 0.50x$. Find

(i) The mean of x.

(ii) The mean of y.

(iii) The correlation coefficient of x and y.

B) Given $U = a + bX$ and $W = c + dY$, where a, b, c, d are constants; then prove that $\rho_{UW} = \pm\rho_{XY}$ (3)

C) Fit a straight line for the following data: (3)

x	1	2	3	4	6	8
y	2.4	3	3.6	4	5	6

4) Using Simplex method, solve the LPP (4)

A) Maximize $Z = 5x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0.$

B) Solve using graphical method, (3)

Maximize $Z = 10x_1 + 20x_2$

Subject to $3x_1 + 4x_2 \leq 30$

$5x_1 + 2x_2 \leq 20$

$x_1, x_2 \geq 0.$

C) Solve the difference equation $y_{n+2} - 2\cos\alpha\, y_{n+1} + y_n = \cos \alpha n.$ (3)

5) The height of 500 soldiers are found to have normal distribution. Of them 258 are found to be within 2cm of the mean height of 170cm. Find the standard deviation of X (4)

A)

B) In playing with an opponent of equal ability which is more probable? (3)

i. Winning 3 games out of 4 or winning 5 games out of 8.

ii. Winning atleast 3 games out of 4 or winning atleast 5 games out of 8.

C) Solve the difference equation $y_{n+2} - 4y_n = n^2 + n - 1.$ (3)

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