Exam Date & Time: 30-Apr-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH END SEMESTER EXAMINATIONS, APRIL 2024

ENGINEERING MATHEMATICS - IV [MAT 2224]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates:Missing data may be suitably assumed

1) Solve the ODE $y'' + x^2y = 0,\ y(0) = 0,\ y(1) = 1; h = 0.25$ using finite difference method. (4)

A)

B) With h = $\frac{1}{3}$, Solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0;\ 0 < x < 1; 0 < y < 1; u(x,1) = u(0,y) = 0;$$

$$u(1,y) = 9(y - y^2);\ u(x,0) = 9(x - x^2)$$ (3)

using finite difference method.

C) Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; 0 < x < 1; t > 0; h = 0.25 ; $u(x,0) = 100(x - x^2)$; $u(0,t) = u(1,t) = 0$ up to 2 levels by Crank Nicolson Method. (3)

2) If X_1, X_2, X_3 are uncorrelated random variables having same standard deviation. Find the correlation coefficient between $U = X_1 + X_2$ and $W = X_2 + X_3$. (4)

A)

B) Suppose that an office has 100 calculating machines. Some of these machines are electric while others are manual. Some of these are new while others are used. There are 40 new electric machines and 30 new manual machines. Also, there are 20 used electric machines and 10 used manual machines. A person enters the office, picks a machine at random and discovers that it is new. What is the probability that it is electric? (3)

C) An alloy contains certain percentage of lead say X, which may be considered as a pdf (3)

$$f(x) = \begin{cases} \frac{3}{5}[10^{-5}x(100-x)]; & 0 \leq x \leq 100 \\ 0; & elsewhere \end{cases}$$

Suppose the net profit realized in selling this alloy is P = $C_1 + C_2X$. Compute the expected profit per pound.

3) Find the two regression lines for the following data

A)

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

(4)

B) Suppose that the two-dimensional random variable (X, Y) has joint pdf

$$f(x) = \begin{cases} k(x+y-2xy); & 0<x<1, \quad 0<y<1 \\ 0; & elsewhere \end{cases}$$

(3)

a. Evaluate k.

b. Find the marginal pdf of X and Y.

C) Fit a second degree parabola $y = a + bx + cx^2$ to the following data

x	1	3	5	7	9
y	2	7	10	11	9

(3)

4) Solve using simplex method,

A)

$$Maximize\ Z = 10x + y$$

$$Subject\ to \quad x + y \leq 10$$

$$4x + y \leq 20$$

$$x + 2y \leq 30$$

$$x, y \geq 0$$

(4)

B) Solve using graphical method,

(3)

$$Maximize\ Z = 40x + 80y$$

$$Subject\ to\ \ 2x + 3y \leq 48$$

$$x \leq 15$$

$$y \leq 10$$

$$x, y \geq 0$$

C) Solve $u_{n+2} + 3u_{n+1} + u_n = n\,2^n$ (3)

5) In a certain examination the percentage of passes and distinction were 46 and 9 respectively. Estimate the average marks obtained by the candidates , the minimum pass and distinction marks being 40 and 75 respectively. Assume the distribution of marks to be normal. (4)

A)

B) The probability that a news reader commits no mistakes in reading news is $\frac{1}{e^3}$. Find the probability that on a particular news broadcast he commits

i. Only 2 mistakes

ii. More than 3 mistakes (3)

iii. Atmost 3 mistakes.

C) Solve $(E^2 - 2E + 2)y_n = \cos\frac{n\pi}{2}$ (3)

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