Question Paper

Exam Date & Time: 03-May-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester B.Tech (AIML- Comp.Sc) Engineering
End semester examination

PROBABILITY AND OPTIMIZATION [MAT 2233]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions. Section Duration: 180 mins

Missing data if any can be assumed with proper reasoning.

1a) A committee of `k` people has to be chosen from a set of 7 women and 4 men. How many ways are there to form the committee if (4)
a) the committee has 5 people of which 3 are women and 2 are men.
b) the committee can be of any positive size, but must have equal number of men and women.
c) the committee has four people and one of them must be Mr. X.
d) the committee has four people and at least two of them are women.

1b) How many ways three integers be selected from 3n consecutive integers so that the sum is a multiple of 3? (3)

1c) Use generating function to count all selection of 6 objects from 3 types of objects with repetitions up to 4 times of each type. (3)

2a) Box 1 contains 4 black and 5 green balls and box 2 contains 5 black and 4 green balls. 3 balls are randomly drawn from box 1 without replacement and transferred to box 2 and then a ball is drawn from box 2 and is found to be green. What is the probability that 2 green and 1 black balls are transferred from box 1? (4)

2b) One number is chosen at random from 1,2,3,4,..., 50. Find the probability that chosen number is divisible by 6 or by 8. (3)

2c) If a random variable K is uniformly distributed over (0,5), what is the probability that the roots of the equation $4x^2 + 4xk + k + 2 = 0$ are real? (3)

3a) The probability of a man hitting a target is $\frac{1}{3}$. (4)
(a) If he fires 5 times, what is the probability of his hitting the target atleast twice?
(b) How many times must he fires so that the probability of his hitting the target atleast once is more than 90%.

3b) Suppose that the continuous random variable X has probability density function $f(x) = e^{-x}$, $x > 0$. Find the probability density function of $Y = X^3$. (3)

3c) (3)

If $f(x,y)$ is uniformly distributed over R, where R is the region bounded by $y = x$, $y = x^2$. Find the marginal probability distribution of X and Y.

4a) The monthly income of 10000 persons is found to be normally distributed if the mean salary is Rs. 7500 and standard deviation is Rs.500. (3)
(a)What percentage of the persons have income exceeding Rs. 6680?
(b)What is the lowest income among the richest 100?

4b) Find the directional derivative of $\emptyset = x^2yz$ at $(-1,1,3)$ in a direction towards the point $(-3,5,6)$. (5)

4c) Find the constants a and b so that $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1,-1,2)$. (2)

5a) Maximize $Z = 5x_1 + 3x_2$ (4)
Subject to $x_1 + x_2 \le 2$, $5x_1 + 2x_2 \le 10$, $3x_1 + 8x_2 \le 12$,
$x_1 \ge 0$, $x_2 \ge 0$ using simplex method.

5b) Maximize $Z = 2x_1 + 3x_2$ (3)
subject to $x_1 + x_2 \le 30$, $x_1 - x_2 \ge 0$, $x_2 \ge 3$, $0 \le x_2 \le 12$, $0 \le x_1 \le 20$
using graphical method.

5c) A company produces three products A, B, C. For manufacturing three raw materials P, Q and R are used. (3)
Profit per unit : A - Rs. 5, B - Rs. 3, C - Rs. 4
Resource requirements/unit

Raw Material / Product	P	Q	R
A	-	20	50

B	20	30	-
C	30	20	40

Maximum raw material availability: P - 80 units; Q - 100 units; R - 150 units.
Formulate LPP.

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