

Question Paper

Exam Date & Time: 05-May-2024 (02:30 PM - 05:30 PM)



MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH. (ELECTRONICS AND COMMUNICATION ENGINEERING) DEGREE EXAMINATIONS - APRIL / MAY 2024
SUBJECT: MAT 2227-ENGINEERING MATHEMATICS - IV

Marks: 50

Duration: 180 mins.

Answer all the questions.

1A) The chances that doctor A will diagnose a disease X correctly is 60 percent. The chances that a patient will die by his treatment after correct diagnosis is 40 percent and the chance of death by wrong diagnosis is 70 percent. A patient of doctor A, who had disease X, died. What is the chance that his disease was diagnosed correctly. (3)

1B) A woman has n keys, of which one will open her door. If she tries the keys at random, discarding those that do not work, what is the probability that she will open the door on her k th try? What if she does not discard previously tried keys? (3)

1C) A bag contains 3 white marbles and 2 red marbles, marbles are drawn from the bag randomly one after the other (without replacement) until all the white marbles are drawn. Let X denote the number of draws required to get the first white marble and Y denote the number of additional draws required to get the second white marble. find (4)
i) joint probability distribution of X and Y.
ii) marginal probability mass function of X and Y, respectively.
iii) $P(X \leq 2 \cap Y \geq 2)$.

2A) The density function of X is given $f(x) = \begin{cases} a + bx^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$. If $E(X) = \frac{3}{5}$, find a and b. (3)

2B) Suppose X and Y are two random variables such that $Y = aX + b$. Then show that the correlation coefficient between X and Y is ± 1 . (3)

2C) The annual rainfall (in inches) in a certain region is normally distributed with mean 40 and standard deviation 4. What is the probability that in 2 of next 4 years the rainfall will exceed 50 inches? (4)

3A) i) The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years? (4)
ii) You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30. What is the probability that you will have to wait longer than 10 minutes?

3B) Suppose $f(x) = \frac{e^{-|x|}}{2}$, $-\infty < x < \infty$ is the density function of X. Find the moment generating function of X and hence find its mean (3)

3C) If $X \sim N(0, \sigma^2)$ and $Y \sim N(0, \sigma^2)$ are independent random variables, obtain the density function of $\sqrt{X^2 + Y^2}$. (3)

4A) A factory has 2 machines and one repair crew. Assume that probability of any one machine breaking down on a given day is $\frac{1}{2}$. Assume that if the repair crew is working on a machine, the probability that will complete the repair in a day is $\frac{1}{2}$. For a simplicity, ignore the probability of repair completion or breakdown taking place except at the end of the day. Let X_n be the number of machines in operation at the end of the n th day. Assume the behaviour of X_n can be modeled as a Markov chain. Find the transition probability matrix. (3)

4B) Solve the following LPP by graphical method. (3)

$$\text{Minimize } Z = 4x + y$$

Subject to,

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, \quad y \geq 0$$

4C)

Use the Simplex method to solve the following LPP. (4)

$$\text{Max } Z = 16x_1 + 17x_2 + 10x_3$$

Subject to

$$x_1 + 2x_2 + 4x_3 \leq 2000$$

$$2x_1 + x_2 + x_3 \leq 3600$$

$$x_1 + 2x_2 + 2x_3 \leq 2400$$

$$x_1 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

5A)

Iterate for 2 steps to find extremum of $f(x) = x_1^2 + 3x_2^2$, starting from $(6, 3)$ using the method of steepest descent. (4)

5B)

If $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$, find the values of a, b, c such that $\text{curl}(\vec{F}) = 0$. (3)

5C)

Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along the direction $2i - 3j - 6k$. (3)

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