Question Paper

Exam Date & Time: 21-Jun-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH. (ELECTRONICS AND COMMUNICATION ENGINEERING) DEGREE EXAMINATIONS - JUNE 2024
SUBJECT: MAT 2227/MAT_2227 - ENGINEERING MATHEMATICS - IV

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) In a certain college, 4% of the boys and 1% of the girls are taller than 1.8m. Furthermore, 60% of the students are girls. If a student is chosen at random and is found to be taller than 1.8m, what is the probability that the student is a girl. (3)

1B) The odds that person X speaks the truth are 3:2 and the odds that person Y speaks the truth are 5:3. In what percentage of cases are they likely to contradict to each other on an identical point? (3)

'C) (4)

If $f(x,y) = \begin{cases} kxy, & 0 < x < y < 2 \\ 0, & otherwise \end{cases}$ is the joint density function of the random variable (X, Y), then find the value of k and also determine the marginal density function of X and Y.

2A) A continuous random variable X has the cumulative distribution function (3)

$F(x) = \begin{cases} 0, & x \leq 1 \\ k(x-1)^4, & 1 < x \leq 3 \\ 1, & x > 3 \end{cases}$. Find (i) k. (ii) density function of X. (iii) the mean of X.

2B) If X, Y and Z are uncorrelated random variables with standard deviation 5, 12 and 9, respectively. Evaluate the correlation coefficient between $U = 3X + Y$ and $V = Y + 4Z$. (3)

C) (4)

Let X be a normal variate such that $P(X < 45) = 0.31$ and $P(X > 64) = 0.48$. Compute the mean and variance of X.

3A) It is known that disks produced by a certain company will be defective with probability .01 independently of each other. The company sells the disks in packages of 10 and offers a money-back guarantee that at most 1 of the 10 disks is defective. What proportion of packages is returned? If someone buys three packages, what is the probability that exactly one of them will be returned? (3)

3B) In an experiment of tossing a fair dice, let X be the number of tosses required to get any one of the numbers 2, 4, or 6. Find the moment generating function of X and hence find its mean. (3)

3C) (4)

Suppose that X and Y are two independent random variables having density function $e^{-x}, 0 < x < \infty$ and $4e^{-4y}, 0 < y < \infty$. Find the density function of X+2Y.

4A) An Urn initially contains 5 black balls and 5 white balls. The following experiment is repeated indefinitely. A ball is drawn from the Urn, if the ball is white, it is put back in the urn otherwise it is left out. Let X_n be the number of black balls remaining in the urn after n draws from the urn. Find the Transition probability matrix. (3)

4B) Solve the following LPP by graphical method. (3)

Maximize $Z = 4x + y$

Subject to,

$$x + y \leq 50$$
$$3x + y \leq 90$$

$$x \geq 0, \quad y \geq 0$$

4C) Use the Simplex method to solve the following LPP. (4)

Max $Z = 4x_1 - x_2$

Subject to

$$x_1 + 2x_2 \leq 4$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

5A) Iterate for 2 steps to find extremum of $f(x) = x_1^2 + 4x_2^2 - 6x_1 - 8x_2$, starting from the point $(4, 0)$ using the method of steepest descent. (4)

5B) Prove that $div(curl\, \vec{A}) = 0$. (3)

5C) Find the directional derivative of the equation $\emptyset = x^2yz + 4xz^2$ at $(1, -2, -1)$ along the direction $2i - j - 2k$. (3)

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