EIC, CPS, ICE, EIE

Exam Date & Time: 21-Jun-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester Makeup End Semester Examination

ENGINEERING MATHEMATICS - IV [MAT 2228]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) A) The probability mass function of the random variable is

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find i) $P(X \le 4)$ ii) $P(X \ge 5)$ iii) $P(3 < X \le 6)$. (3)

B) Let a random variable X be uniformly distributed in the interval $[-5,5]$. Then find

i) $P(X < 2)$ ii) $P(X > -3)$ iii) $P(|X| < 1)$. (3)

C) Box 1 contains 4 black and 5 green marbles and box 2 contains 5 black and 4 green marbles. The 3 marbles are selected at random from box 1 and transferred to box 2, then a marble is selected at random from box 2. What is the probability that this marble is green? If this marble is green, what is the probability that 2 green and one black marble are transferred from box 1 to box2 ? (4)

2) A) Suppose that X has pdf $f(x) = \begin{cases} 2e^{-2x}, & x \ge 0 \\ 0 & otherwise \end{cases}$. Then find the moment generating function (mgf) of X and using mgf find the $E(X)$. (3)

B) In the normal distribution, *31%* of items are less than *45* and *8%* of items are over *64*. Find the mean and standard deviation of the distribution. (3)

C) A saleman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in other city. In the long run, how often does he sell in each of the cities. (4)

3) If *X, Y, Z* are uncorrelated random variables with standard deviation *5, 12* and *9* respectively. If $U = X + Y$ and $V = Y + Z$, then evaluate the correlation coefficient between *U* and *V*. (3)

A)

B) The joint probability density function of the random variable (X,Y) is given by

$$f(x,y)=\begin{cases} kx(x-y); & 0<x<2,\ -x<y<x \\ 0; & elsewhere \end{cases}$$ (3)

a. Find k . (b) Show that X and Y are independent random variables.

C) Let X be a random variable having pdf $f(x)=\begin{cases} 1; & 0\le x\le 1 \\ 0; & elsewhere \end{cases}$. Find the pdf of $Y=-2logX$. (4)

4) If $P(A)=\frac{1}{3}$ and $P(A\cup B)=\frac{1}{2}$, find $P(\bar{B})$ and P $(\bar{A}|\bar{B})$ in the following cases.

A)
i. *If* *A* and *B* are mutually exclusive. (3)

ii. *If* *A* and *B* are independent.

B) Find the directional derivative of $\phi=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction towards the point $(1,2,2)$. (3)

C) Show that $\vec{A}=(x^2-y^2+x)\,\hat{\imath}-(2xy+y)\,\hat{\jmath}$ is irrotational. Also find a scalar potential (or function) such that $\vec{A}=\nabla\phi$. (4)

5) Solve by graphical method

A)
$Max\ Z=5x_1+3x_2$ subject to the constraints

$4x_1+5x_2\le 1000, 5x_1+2x_2\le 1000, 3x_1+8x_2\le 1200$ with $x_1,x_2\ge 0.$ (3)

B) Solve by simplex method

$Max\ Z=3x_1+4x_2$ subject to the constraints

$x_1+x_2\le 450, 2x_1+x_2\le 600$ with $x_1,x_2\ge 0.$ (3)

C) Solve the nonlinear programming problem.

$Optimize\ Z=4x_1^2+2x_2^2+x_3^2-4x_1x_2$ subject to the constraints

$x_1+x_2+x_3=15, 2x_1-x_2+2x_3=15$ with $x_1,x_2,x_3\ge 0.$ (4)