END SEMESTER EXAMINATION-APRIL MAY 2024

Subject: ENGINEERING MATHEMATICS-IV_MAT 2230

Branch: Mechanical Engineering

<table>
<tr><th>Q. No.</th><th>Questions</th><th>Marks</th><th>CO</th><th>BT</th></tr>
<tr><td>1A</td><td>Fit a parabola $y = ax^2 + bx + c$ to the following data<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>2.1</td><td>7.7</td><td>13.6</td><td>27.2</td><td>40.9</td><td>61.1</td></tr></table></td><td>4</td><td>CO1</td><td>3</td></tr>
<tr><td>1B</td><td>If A and B are two independent events of S, such that $P(\bar{A} \cap B) = \frac{2}{15}$, $P(A \cap \bar{B}) = \frac{1}{6}$, then find $P(B)$.</td><td>3</td><td>CO1</td><td>3</td></tr>
<tr><td>1C</td><td>A continuous random variable X has a pdf $f(x) = k\, e^{-|x|}, -\infty < x < \infty$. Then evaluate (i) The constant 'k' (ii) Mean $E(X)$ and (iii) Variance $V(X)$.</td><td>3</td><td>CO2</td><td>4</td></tr>
<tr><td>2A</td><td>A bag contains five balls and it is not known how many of them are white. Two balls are drawn from the bag at random and they are noted to be white. What is the chance that all the balls in the bag are white?</td><td>4</td><td>CO1</td><td>4</td></tr>
<tr><td>2B</td><td>Two independent random variable X and Y have mean 7 and 11 and variance 100 and 196 respectively. Find the correlation coefficient between M=3X+4Y and N = X+5Y.</td><td>3</td><td>CO2</td><td>3</td></tr>
<tr><td>2C</td><td>A two dimensional random variables (X, Y) has a joint probability mass function $p(x, y) = \frac{1}{27}(2x + y)$, where x and y can assume only the integer values 0, 1, 2. Compute $P[(X + Y) \leq 3]$ also find the standard deviation of X.</td><td>3</td><td>CO2</td><td>4</td></tr>
<tr><td>3A</td><td>If X and Y are two random variables having joint pdf $f(x, y) = \begin{cases} k(6 - x - y), & 0 \leq x \leq 2;\ 2 \leq y \leq 4 \\ 0, & elsewhere \end{cases}$. Then find (i)The value of k
(ii) Marginal pdf of X
(iii) Find the conditional pdf of Y given X
(iv) $P(X > 1)$</td><td>4</td><td>CO2</td><td>4</td></tr>
<tr><td>3B</td><td>The marks of 1000 students in an examination forms a Normal distribution with mean 70 and</td><td>3</td><td>CO3</td><td>3</td></tr>
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	standard deviation 5. Find the number of students whose marks will be (i) Less than 65 (ii) Greater than 75 (iii) Between 65 and 75			
3C	Suppose that X is uniformly distributed random variable in the interval $(-1, 1)$. Obtain the pdf of the random variable $Y = sin\left(\frac{\pi X}{2}\right)$.	3	CO4	4
4A	Derive the mean and variance of Gamma distribution.	4	CO3	3
4B	If X and Y are independent random variables having moment generating functions $M_X(t) = e^{3t+\frac{9t^2}{2}}$ and $M_Y(t) = e^{5t+\frac{3t^2}{2}}$. Find the moment generating function (Mgf) of $Z = 3X + 4Y$. Also find $E\left(\frac{Z}{3}\right)$ and $V\left(\frac{Z}{3}\right)$.	3	CO4	4
4C	Let $\bar{X}$ and S^2 be the mean and variance of a random sample of size 25 from a distribution $N(3, 100)$. Evaluate the following (i) $P(0 < \bar{X} < 6)$ (ii) $P(55.2 < S^2 < 145.6)$ (iii) $P(0 < \bar{X} < 6,\ 55.2 < S^2 < 145.6)$	3	CO4	4
5A	Obtain the series solution of the differential equation $4xy'' + 2y' + y = 0$ by Frobenius method.	4	CO5	4
5B	Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, sinx$	3	CO5	3
5C	A computer in adding numbers, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over $(-0.5, 0.5)$. If 1500 numbers are added, then what is the probability that the magnitude of the total error exceeds 15?	3	CO4	4