Question Paper

Exam Date & Time: 19-Jun-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH. (INFORMATION TECHNOLOGY) DEGREE EXAMINATIONS - JUNE 2024
SUBJECT: MAT 2226/MAT_2226 - ENGINEERING MATHEMATICS - IV

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

Missing data may be suitably assumed.

1A) Six people toss a fair coin one after the other until a player obtains a head. The game is won by the player who throws head first. Find the probability of success of the 4th player. (4)

1B) Two defective tubes get mixed up with 6 good ones. The tubes are tested one by one, until both defective tubes are found. What is the probability that the last defective tube is obtained on (a) the 2nd test, (b) the 3rd test. (3)

1C) Let X be a discrete random variable having the probability mass function given by: (3)

x	1	2	3	4	5
P(X = x)	1/15	2/15	3/15	4/15	5/15

(i) Check whether the given function is a valid probability mass function.
(ii) Calculate P(1/2 < X < 5/2 | X > 1).

2A) Find the mean and variance of a random variable X which is uniformly distributed in the interval (-a, a). (4)

2B) An aircraft knows that 5% of the people making reservation on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for the flight that can only hold 50 passengers. What is the probability that there will be a seat available for every passenger who turns up? (3)

2C) Let X has binomial distribution with parameter n=10 and $p \in \{p : p = \frac{1}{4}, \frac{1}{2}\}$. $H_0: p = \frac{1}{2}$. Is rejected and $H_1: p = \frac{1}{4}$ is accepted. If the observed values of X_1 a random sample of size 1 is less than or equal to 3, find the power function of the test. (3)

3A) Let (X, Y) have the joint pdf (4)

$$f(x,y) = \begin{cases} 2y, & 0 < x < 1, 0 < y < 1 \\ 0 & otherwise. \end{cases}$$

(i) Find the mean and variance of X and Y.
(ii) Find E(X | Y).

3B) Urn A contains 5 black balls, 6 white balls. Urn B contains 8 black balls and 4 white balls. Two balls are transferred from B to A and then a ball is drawn from A. (3)
i) What is the probability that this ball is white?
ii) Given that the ball is drawn is white. what is the probability that one black ball was transferred to A?

3C) The survival time X (in terms of weeks) of a male mouse exposed to radiation has Gamma distribution with PDF (3)

$$f(x) = \begin{cases} \left(\frac{1}{15}\right)^3 \frac{x^2 e^{-\frac{x}{15}}}{\Gamma(3)}, & x > 0 \\ 0, & otherwise \end{cases}$$

Find the probability that a mouse survives:
i) Between 60 and 120 weeks
ii) At least 30 weeks

4A) Suppose X and Y are independent random variables. X takes values 2, 5, 7 with probability $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ respectively. Y takes the value 3, 5 with probability $\frac{1}{3}, \frac{2}{3}$. (4)
i) Find joint probability distribution of X and Y.
ii) Find probability distribution of Z = X + Y.

4B) The probability that a news reader commits no mistakes in reading the news during one broadcast is $\frac{1}{e^3}$. Find the probability that in ten news broadcasts he commits (3)
i) Only one mistake
ii) More than 3 mistakes
iii) At most 2 mistakes.

4C) Assume that the average life span of computers produced by the company is 2040 hours with standard deviation 60 hours. Find the expected number of computers, whose life span is: (3)
i) more than 2150 hours,
ii) less than 1950 hours,
iii) between 1920 and 2160 hours
from a pool of 2000 computers, assuming that life span, X, is normally distributed.

5A) According to the ideal grading curve, the percentages of students receiving grades A+, A, B, C, D, E, and F should be, respectively, 0.1, 2.2, 13.6, 34.1, 34.1, 13.6, and 2.3. In an exam with 2000 students, the numbers of students getting grades A+, A, B, C, D, E, and F are, respectively, 3, 48, 290, 642, 724, 280, 13. Test whether the grade distribution fits the ideal grading curve, using Chi-square test at 5% significance level. (4)

5B) Let $(X_1, X_2, \ldots, X_n)$ be a random sample from normal distribution $N(0, \theta)$, where $0 < \theta < \infty$. Show that $Y = \frac{\sum_{i=1}^{n} X_i^2}{n}$ is an unbiased and consistent estimator for θ. (3)

5C) The mean life length of a certain cutting tool is 41.5 hours with a standard deviation of 2.5 hours. What is the probability that a random sample of size 50 drawn from this population will have a sample mean between 40.5 and 42 hours. (3)

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