Exam Date & Time: 14-Jun-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FOURTH SEMESTER B.TECH END SEMESTER MAKEUP EXAMINATIONS, JUNE 2024

ENGINEERING MATHEMATICS - IV [MAT 2231]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Missing data may be suitably assumed

1) Given $f(x) = \begin{cases} kx^3; & 0 < x < 1 \\ 0; & otherwise \end{cases}$

A)

i. Find k such that $f(x)$ is a valid pdf

ii. Find the expression for CDF (3)

iii. Find $P\left(\frac{1}{4} < X < \frac{3}{4}\right)$

B) If $f(x) = \begin{cases} 2x; & 0 < x < 1 \\ 0; & Otherwise \end{cases}$, and $Y = e^{-X}$, then find pdf of Y (3)

C) The random variables (X, Y) have a joint pdf given by

$$f(x,y) = \begin{cases} x + y; & 0 < x < 1,\ 0 < y < 1 \\ 0 & Otherwise \end{cases}$$ (4)

Find the correlation coefficient between X and Y

2) (3)

A) Let $f(x,y) = \begin{cases} \frac{xy}{96}; & 0 < x < 4,\ 1 < y < 5 \\ 0; & Otherwise \end{cases}$

i. Find marginal pdf of X and Y

ii. Check whether X and Y are independent? Give a reason

iii. Find $P(1 < X < 2,\ 2 < Y < 3)$

B) Player X and Y roll a pair of dice alternatively. The player who rolls sum equal to 11 first, wins the game. If player X starts the game, then find his chance of winning (3)

C) $$u_{n+2} - 4u_{n+1} + 4u_n = 2^n$$

Solve the difference equation: (4)

3) Fit a curve of the form $y = ab^x$ to the following data.

A)

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

(3)

Hence find the value of y when x=8

B) An insurance company has discovered that only about 0.1% of the population is involved in a certain type of accident each year. If its 10,000 policy holders were randomly selected from the population, what is the probability that not more than 5 of its clients are involved in such an accident next year? (3)

C) Local authorities in a certain city install 10000 electric lamps in streets of the city. If these lamps have an average life span of 1000 burning hours with a standard deviation of 200 hours and following normal distribution, what is the number of lamps might be expected to fail (4)

i. in first 800 hours

ii. between 800 and 1200 hours

4) Let $\overline{X}$ be the mean of a sample of size 5 from a normal distribution with mean $\mu = 0$,

A) variance 125. Find C so that $P(\overline{X} < C) = 0.9$. (3)

B) Two independent random variables, X and Y have distributions $N(45, 2^2)$ and $N(44, 1.5^2)$. Obtain $P(|X-Y| \geq 1.5)$ (3)

C) Obtain the series solution of $y'' + xy = 0$ (4)

5) Solve by Z-Transform:

$$y_{n+2} - 6y_{n+1} + 8y_n = 4^n,\ y_0 = 0,\ y_1 = 1$$ (3)

A)

B) Prove the following

i. $$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$$

ii. $$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$$ (3)

C) A Laboratory test is 95% efficient in detecting a certain disease when it is present. Moreover, the test also yields false positive for 1% of the healthy person tested. If 0.5% of the population has the disease, what is the probability that a person has the disease, given that the test result in positive? (4)

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