MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEMESTER M. TECH (Computer Science & Engineering)
END SEMESTER MAKEUP EXAMINATION, January 10, 2024
SUBJECT: QUANTUM COMPUTING (CSE 5115)
REVISED CREDIT SYSTEM

Time: 3 Hours (9.30 AM-12.30 AM) **MAX. MARKS: 50**
Note: Answer ALL the questions.

1A	A qubit is prepared in state $\lvert\psi\rangle = \frac{1}{\sqrt{2}}\lvert+\rangle + \frac{i}{\sqrt{2}}\lvert-\rangle$. It is then measured. Compute P(+) and P(-)	2
1B	Suppose $\lvert\psi\rangle = \frac{1}{\sqrt{2}}(\lvert 0\rangle + \lvert 1\rangle)\, and \,\lvert\varphi\rangle = \frac{1}{\sqrt{2}}(\lvert 0\rangle - \lvert 1\rangle)$. Compute $\langle\psi\vert\varphi\rangle$	3
1C	Determine whether the following two qubit state is entangled or not. Justify your answer. $\frac{\lvert 00\rangle - \lvert 11\rangle}{\sqrt{2}}$	5
2A	Let $\lvert\psi_1\rangle = \frac{1}{\sqrt{3}}\lvert 0\rangle + \sqrt{\frac{2}{3}}\lvert 1\rangle\, and \,\lvert\psi_2\rangle = \frac{1}{2}\lvert 0\rangle + \frac{\sqrt{3}}{2}\lvert 1\rangle$ Compute $\lvert\psi_1\rangle \otimes \lvert\psi_2\rangle$	3
2B	Let H is Hadamard and $\lvert\psi\rangle = \frac{1}{2}(\lvert 00\rangle + \lvert 01\rangle + \lvert 10\rangle + \lvert 11\rangle)$. Compute $\lvert\psi_1\rangle = (H \otimes H)\lvert\psi\rangle$	4
2C	Define CCNOT gate. Design quantum circuit for CCNOT gate and give its matrix representation. With circuit diagram Implement FANOUT operation using CCNOT gate.	3
3A	Define Y Gate. Derive the Braket representation of Y gate.	3
3B	Compute the matrix for the three-qubit Toffoli gate.	2
3C	Compute the matrix for 2 qubit QFT.	5
4A	Let $f : \{0,1\}^2 \to \{0,1\}$ be a binary function. Consider the quantum state $\lvert\psi_0\rangle = \lvert 0\rangle^{\otimes 2} \otimes \lvert 1\rangle$ (i) Apply H gate to each qubit of $\lvert\psi_0\rangle$. Let $\lvert\psi_1\rangle = H\lvert\psi_0\rangle$ (ii) Apply Oracle $U_f : \lvert x, y\rangle \to \lvert x, y \oplus f(x)\rangle$ to $\lvert\psi_1\rangle$. Let $\lvert\psi_2\rangle = U_f\lvert\psi_1\rangle$. (iii) Apply H gate on the first 2 qubits of $\lvert\psi_2\rangle$.	5
4B	Alice wishes to send Bob a message via a secure protocol. She chooses to use a private key encryption technique and decides to use the BB84 protocol to generate a provably secure private encryption key. Alice's first step is to generate a random binary string. The string she generates is *b* = 1001011110101000. Alice then encodes this as a string of quantum qubits as per the BB84 protocol, using the encoding H H H I I H I H H I I I H I I H. What is the quantum string that she generates? Alice then sends the string to Bob, who decodes using I H H I H I H H I H I I I H I H. Alice and Bob announce their encodings publicly and retain the qubits for which they chose the same encoding. Compute the string that they retain?	3

4C	Suppose Alice transmits the two-bit string ‘00’ using the superdense coding protocol and an evesdropper, Charlie, intercepts the qubit transmitted by Alice, measures it and then re-transmits to Bob. Calculate the probability that Bob correctly receives ‘00’.	**2**
5A	Using quantum algorithm compute the period of the function $f(x) = 11^x \bmod 15$	**5**
5B	Explain 3 challenges in quantum error correcting codes.	**2**
5C	Imagine we encode the state $\vert\Psi> = \alpha\vert 0> + \beta\vert 1>$ using Shor’s 9 qubit code, and then an X error occurs on the 8th qubit of the encoded state $E\vert\Psi>$. Determine the state following the error.	**3**