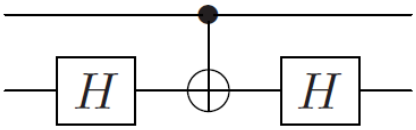
MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

I SEMESTER M. TECH (Computer Science & Information Security)
END SEMESTER EXAMINATION, May 9, 2024
SUBJECT: INTRODUCTION TO QUANTUM COMPUTING (CSE 5420)
REVISED CREDIT SYSTEM

Time: 3 Hours (9.30 AM-12.30 AM) **MAX. MARKS: 50**

Note: Answer ALL the questions.

1A	Show that $\|0\rangle = \frac{1}{\sqrt{2}}(\|+\rangle + \|-\rangle)$ *and* $\|1\rangle = \frac{1}{\sqrt{2}}(\|+\rangle - \|-\rangle)$.	**2**
1B	Prove that $\frac{1}{\sqrt{2}}(\|00\rangle + \|11\rangle) = \frac{1}{\sqrt{2}}(\|++\rangle + \|--\rangle)$	**4**
1C	Given two qubits in the state $\|\psi\rangle = \frac{1}{\sqrt{2}}(\|++\rangle + \|--\rangle)$, Compute the probabilities of measuring , $\|++\rangle, \|+-\rangle, \|-+\rangle$ *and* $\|--\rangle$.	**4**
2A	Consider the following two qubit quantum circuit. (H is Hadamard gate)  Compute outputs for the inputs $\|00\rangle, \|01\rangle, \|10\rangle$ *and* $\|11\rangle$. Hence compute the matrix representation of the above quantum circuit.	**5**
2B	Let $U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ Prove that U is Unitary. Let $\|\psi\rangle = \left(\frac{1}{2} + \frac{i}{2}\right)\|0\rangle + \left(\frac{1}{2} - \frac{i}{2}\right)\|1\rangle$ Compute $U\|\psi\rangle$.	**3**
2C	With neat diagram, explain Bloch sphere.	**2**
3A	Examine with explicit calculation whether the following two qubit quantum state is entangled or not. $\|\phi\rangle = \frac{1}{\sqrt{6}}(\|00\rangle + i\|01\rangle + 2\|10\rangle)$	**5**
3B	Define Bell state. Construct FOUR standard Bell states.	**3**

3C	With neat circuit diagram define Fredkin gate and give its matrix representation.	**2**
4A	Construct 2 qubit QFT for all inputs and give its matrix representation.	**5**
4B	With quantum circuit implement NAND, and XOR gates using Toffoli gate.	**3**
4C	Suppose Alice transmits the two-bit string '00' using the superdense coding protocol and an evesdropper, Charlie, intercepts the qubit transmitted by Alice, measures it and then re-transmits to Bob. Calculate the probability that Bob correctly receives '00'.	**2**
5A	Using quantum algorithm compute the period of the function $f(x) = 3^x \bmod 20$	**5**
5B	Design the quantum circuit for Deutsch-Jozsa algorithm.	**2**
5C	Imagine we encode the state $\alpha\|0\rangle + \beta\|1\rangle$ using the bit-flip code (i.e. $\|0\rangle \rightarrow \|000\rangle$ and $\|0\rangle \rightarrow \|111\rangle$) and a Y error occurs on the second qubit. Compute the decoded state?	**3**